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PM and AM Noise of Combined Signal Sources

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ABSTRACT - The purpose of this paper is to establish a simple analytically model for computing the PM and AM noise of a signal obtained by combining the output of N, nominally independent, sources that takes into account different levels of noise and signal amplitude. Under ideal conditions, the noise of the combined signal is 1/N of the average source. This model shows that phase deviations between the combined signal and the individual sources leads to cross coupling between PM and AM noise. We find no plausible evidence for an intrinsic dependence of the noise reduction on spectral noise type in linear systems. This analysis assumes that environmental induced AM and PM noise are less than 1/3N below the noise in the individual sources and hence can be ignored.

I. INTRODUCTION

The purpose of this discussion is to lay out a framework i.e. a model that permits one to easily compute the PM noise of a signal obtained by combining the output of multiple sources. The same approach can be used to compute the AM noise of a signal obtained by combining the output of multiple sources. The attraction of considering this approach is that potentially the PM and AM noise of the output signal could be reduced from that of the individual contributions by 10 Log N, where N is the number of nominally equal sources combined. The model presented here allows one to compute the PM even when the sources are of unequal PM noise and of unequal power. It also allows one to take into account the limitations due to phase deviation between the sources originating from static and noise conditions.

II. SIGNAL MODEL FOR COMBINED SOURCES

The definition of the power spectral density (PSD) of phase fluctuations $S_\phi(f)$ is given by

$$S_\phi(f) = \frac{\delta\phi^2(f)}{BW}, \ \text{rad}^2\text{/Hz,or}$$

$$L(f) = \frac{1}{2} S_\phi(f) \equiv \frac{1}{2} \text{PSD}[\phi(f)] \qquad (1)$$

where $\delta\phi^2(f)$ is the mean squared phase fluctuation measured at a Fourier frequency offset from the carrier in a noise bandwidth of BW Hz [1,2]. This includes both the contribution of the upper and lower sideband, which contribute equally [3].

In the case that the integrated RMS phase modulation $\Theta_n(f)$ due to the PM noise is small compared to 0.1 rad, L(f) can be interpreted as ½ the ratio of PM noise power per Hz $[V_n^{PM}(f)]^2/(RBW)$ to the total carrier power V_o^2/R, where R is the source and load impedance. One half of the PM noise power goes into the upper and one half goes into the lower sideband.

$$L(f) \cong \frac{1}{2} \frac{[V_n^{PM}(f)]^2}{V_o^2 BW} \ \text{rad}^2\text{/Hz.} \qquad (2)$$

Often L(f) is expressed in a dB form 10 Log L(f) with units of dBc/Hz, which is short hand for dB below the carrier in a 1 Hz noise bandwidth.

A model for the signal coming from the i^{th} signal source is

$$V_i(t) = [Vo_i + \varepsilon_i(t)][\cos[2\pi\nu_o t + \Theta_i + \Phi_{refi}(t) + \varphi_{ni}(t)] \qquad (3)$$

where V_{oi} is the average amplitude and $\varepsilon(t)$ the amplitude fluctuations of the i^{th} source, ν_o is the average frequency, Θ_i is the static phase offset between the i^{th} source and the multiplied PLL reference, $\Phi_{refi}(t)$ is the phase noise transferred to the i^{th} source from the multiplied PLL reference, and $\varphi_{ni}(t)$ is the phase fluctuations of the oscillator.

Using the above voltage signal and the definition of L(f), we can express the PM noise of the i^{th} source as

$$L_i(f) = \frac{1}{2} S_{\phi i}(f) \equiv \frac{1}{2} \text{PSD}[\Theta_i + \Phi_{refi}(f) + \varphi_{ni}(f)]\,. \qquad (4)$$

Since $\Phi_{refi}(f)$ and $\varphi_{ni}(f)$ are not correlated and Θ_i is static, all cross products average to zero and equation (4) can be expressed as

$$L_i(f) = \frac{1}{2} [\text{PSD}[\Phi_{refi}(f) + \text{PSD}[\varphi_{ni}(f)]]\,. \qquad (5)$$

The PSD of fractional amplitude fluctuations $S_a(f)$ is given by

$$S_a(f) = \frac{\delta\varepsilon^2(f)}{BWV_o^2}, \text{ 1/Hz}, \quad (6)$$

where $\delta\varepsilon^2(f)$ is the mean squared amplitude deviation measured at Fourier frequency offset f in a bandwidth of 1 Hz.

The mean phase deviation of the i^{th} source from the phase reference plane defined by the multiplied PLL reference, is given by

$$\Theta_{ni}(f_{BW}) = \sqrt{2\int_{f_{BW}}^{\infty} L_i(f)df}, \quad (7)$$

where f_{BW} is approximately the bandwidth of the PLL. That is, the carrier signal of the i^{th} source fluctuates about the phase of the multiplied reference signal by an amount $\Theta_{ni}(f_{BW})$ due to high frequency noise processes that are not controlled by the PLL.

III. SIMPLE MODEL FOR PM NOISE FROM COMBINED SOURCES

To begin to get some insight into how the noises combine in an ideal case let's assume that $[\Theta_i + \Theta_{ni}(f_{BW})]$ is small compared to 0.1 rad. In this case we can represent the signal by a carrier of amplitude $V_{oi} + V_{ni}^{AM}(f)$ and a PM noise voltage of $V_{refi}^{PM}(f) + V_{ni}^{PM}(f)$, where $V_{refi}^{PM}(f) = V_{oi}\Phi_{refi}(f)$ and $V_{ni}^{PM}(f) = V_{oi}\varphi_{ni}(f)$. $V_{refi}^{PM}(f)$ is the PM noise voltage impressed on the i^{th} source due the PM noise in the multiplied reference by the action of the PLL. $V_{ni}^{PM}(f)$ is the PM noise voltage of the i^{th} source. As an example, let's consider the PM noise obtained from combining three sources that are phase locked together with a bandwidth BW that is very small compared to the offset frequencies f of interest. See Fig. 1. If the three sources are combined in an ideal summer and $V_{refi}^{PM}(f)$ is very small compared to $V_{ni}^{PM}(f)$, then

$$L_{1,2,3}^{Sum} \cong \frac{1}{2}\frac{[V_{ni}^{PM}(f) + V_{n2}^{PM}(f) + V_{n3}^{PM}(f)]^2}{[V_{o1} + V_{o2} + V_{o3}]^2 BW}$$

$$\cong \frac{1}{2}\frac{[V_{ni}^{PM}(f)]^2 + [V_{n2}^{PM}(f)]^2 + [V_{ni3}^{PM}(f)]^2}{[V_{o1} + V_{o2} + V_{o3}]^2 BW}. \quad (8)$$

Since $V_{ni}(f)$ in the various sources are independent, the cross terms average towards zero. For a linear combiner there is no cross-coupling from one offset frequency f to another in the same source, and since the sources are assumed to be independent, there is no coupling of power or phase information between sources except through the PLL. Therefore, these results are independent of the spectral characteristics of the noise. However, the V_{oi} are coherent and are therefore added before squaring. Note that this result is critically dependent on the carrier signals all being in phase to within roughly 0.1 rad. If the carrier signals are not coherent then the output power will be simply a sum of the power in the individual carrier signals and the PM noise will then be an average of the three sources. Therefore, while there certainly are noise processes that show coherency over short times in a *single* source, there is no known theoretical basis for coherency when averaging spectral power over *different* sources at offset frequencies that are large compared to the PLL bandwidth. It is thought that any failures to achieve the results indicated by equations (8) are due to a failure to adequately take into account the PLL effects discussed below.

Equation (8) can easily be generalized to the case of N sources and is independent of the noise type

$$L_{i=1toN}^{Sum} \cong \sum_{i=1}^{N}\left[\frac{V_{oi}(f)}{\sum_{i=1}^{N} V_{oi}}\right]^2 L_i(f) = \sum_{i=1}^{N}\left[\frac{V_{oi}(f)}{V_{Sum}}\right]^2 L_i(f). \quad (9)$$

For the special case with equal amplitudes, $V_{Sum} = N V_{oi}$, this becomes

$$L_{i=1 \text{ to } N}^{Sum} = \frac{1}{N^2}\sum_{i=1}^{N} L_i(f). \quad (10)$$

Note that there is no requirement that the PM noise of each source $L_i(f)$ be equal, or that they have any particular spectral type, for example white PM noise. The small angle restriction used to obtain equation (8) no longer applies. In the case where all $L_i(f) = L_o(f)$, equation (9) simplifies to

$$L_{i=1 \text{ to } N}^{Sum} = \frac{1}{N^2}\sum_{i=1}^{N} L_i(f) = \frac{1}{N} L_o(f). \quad (11)$$

Now let's examine a more realistic case where the PM noise of the various sources are unequal. As an example, let's chose a set where 1/3 of the sources are 3 dB lower than nominal and 1/3 are 3 dB higher than nominal.

$$L_1(f) = 0.5\, L_o(f),\ L_2(f) = L_o(f),\ L_3(f) = 2\, L_o(f). \qquad (12)$$

For the case where the sources are weighted equally, that is V_o is the same for all the sources, the combined PM noise from equation (8) above is equal to

$$L_{1,2,3}^{Sum} \cong \frac{1}{2}\left[\frac{0.5[V_{no}^{PM}(f)]^2 + [V_{no}^{PM}(f)]^2 + 2[V_{no}^{PM}(f)]^2}{[V_o + V_o + V_o]^2\, BW}\right]$$
$$\cong \frac{1}{2}\left[\frac{3.5}{9}\right]\left[\frac{[V_{no}^{PM}(f)]^2}{[V_o]^2\, BW}\right] = \frac{1}{2.57} L_0(f). \quad (13)$$

For the case where the sources are weighted inversely as their PM noise. That is
$V_{oi}(f) = 1.4\, V_o(f), V_{o2}(f) = V_o(f), V_{n3}(f) = 0.707\, V_{no}(f)$
The combined PM noise from equation 7 above is equal to

$$L_{1,2,3}^{Sum} \cong \frac{1}{2}\left[\frac{0.5[1.41V_{no}(f)]^2 + [V_{no}(f)]^2 + 2[0.71V_{no}(f)]^2}{[1.414V_o + V_o + 0.707V_o]^2\, BW}\right]$$
$$\cong \frac{1}{2}\left[\frac{3}{9.74}\right]\left[\frac{[V_{no}^{PM}(f)]^2}{[V_o]^2\, BW}\right] = \frac{1}{3.26} L_0(f). \qquad (14)$$

The maximum difference from the results of equal sources is a mere 0.5 dB. And the difference between the two weighting for the same set of sources is only 1 dB. Therefore there is little concern about variations up to +/- 3 dB in PM noise among the sources contribution to the combined signal or in weighting and there is virtually nothing to be gained by using more complex weighting schemes. Moreover we can use equation (10) to describe the resulting PM noise of the combined signal as long as the variation among the individual sources is nominally less than 3 dB. If the variations are larger than 3 dB then we must resort to Eq. (9).

The above results were obtained assuming that the phase deviation between the outputs of the individual sources and the multiplied PLL reference, $[\Theta_{ni} + \Theta_{ni}(f_{BW})]$, is so small that AM effects can be neglected. This is often not the case in real life. For example, some ultra low PM noise oscillators have AM larger than the PM noise for Fourier offsets from roughly 1 kHz to 50 kHz. An additional effect that needs to be considered in this context is the bandwidth of the PLL. This bandwidth controls the RMS phase deviation coming from PM noise in the sources and also the amount of coherent PM noise impressed on each of the sources by the low frequency reference. Obviously, the coherent part of the PM noise in the individual sources will not average away with multiple sources.

Typically, to prevent the PLL source from limiting the averaging, its contribution should be at least 10 Log 3N lower than the closed loop PM noise of the sources at the lowest offset frequency of interest. This would generally push the PLL bandwidth for the microwave source BWμw to lower values. However, the need to keep the $\Theta_{ni}(f_{BW})$ small may indicate a larger value of BWμw. If these two are not compatible, then it may be necessary to insert an additional reference source to phase lock each independent microwave source at a sufficiently high BWμw so as to keep $\Theta_{ni}(f_{BW})$ manageable. These additional reference sources are then phase locked to the overall all reference at a much lower bandwidth BW_{LF}. This then allows one to greatly reduce the coherent effects of the overall PLL reference. These effects will be explored through example in the PLL section below. See Fig. 2.

IV. EFFECT OF PHASE OFFSETS AND AM NOISE ON THE PM NOISE

We now consider the effect of phase offset between the multiplied PLL reference and the individual microwave sources. If the phase offset is large enough, the AM noise begins to contribute to the PM noise of the summed signals. For this case we find for individual contributions to $L^{Sum}(f)$

$$L_i(f) \cong \frac{[V_{ni}^{PM}(f)]^2 \cos^2[\Theta_i + \Theta_{ni}(f_{BW})]}{2BWV_{oi}^2 \cos^2[\Theta_i + \Theta_{ni}(f_{BW})]} + \frac{[V_{ni}^{AM}(f)]^2 \sin^2[\Theta_i + \Theta_{ni}(f_{BW})]}{2BWV_{oi}^2 \cos^2[\Theta_i + \Theta_{ni}(f_{BW})]}. \qquad (15)$$

So long as the AM noise is small, and the phase deviations are less than 0.5 rad, the phase deviation results primarily in a lower weighting of this source in the combined sum. If, however, the AM noise is 10 dB above the PM noise then $\sin^2[\Theta_{ni} + \Theta_{ni}(f_{BW})]$ needs to be roughly 0.02 or $[\Theta_{ni} + \Theta_{ni}(f_{BW})]$ need to be approximately less than 0.14 rad so as not to increase the PM noise. In some cases this can place a severe requirement on the static phase offset as well BWμw.

For example if $[\Theta_{ni} + \Theta_{ni}(f_{BW})] = 0.3$ rad, then $\sin^2 = 0.09$ and $\cos^2 = 0.91$. So the carrier is not significantly changed (- 0.5 dB). If the AM noise is 10 dB higher that the PM noise, the contribution to the combined PM noise is now doubled, however, due to the AM noise contribution.

V. PLL EFFECTS

We now consider the PM noise of the complete system with PLL effects added to those considered above. PLL effects can be the limiting effect at low offset frequencies. The multiplied reference frequency typically has white PM noise at BWμw, the bandwidth of the servo locking the microwave sources. The PM noise of the microwave source typically is falling as f^{-2} or faster (often f^{-4}) with an offset frequency of BWμw. This typically means that it takes an offset frequency of at least 10 BWμw and often 30 BWμw to reduce the PM noise impressed on the microwave source 20 dB below the uncorrelated contribution. For the PM noise of the combined signals we find

$$L^{\mathrm{Sum}}_{\mathrm{i=1\ to\ N}}(f) \cong \frac{\sum_{i=1}^{N}[\frac{G_i(f)}{1+G_i(f)} V^{PM}_{refi}(f)]^2}{2BW\sum_{i=1}^{N}[V_{oi}\cos[\Theta_i + \Theta_{ni}(f_{BW})]]^2} + \frac{\sum_{i=1}^{N}[\frac{1}{1+G_i(f)} \mathrm{V}^{\mathrm{PM}}_{\mathrm{ni}}(\mathrm{f})\{\cos[\Theta_i + \Theta_{ni}(f_{BW})]\}]^2}{2BW\sum_{i=1}^{N}[V_{oi}\cos[\Theta_i + \Theta_{ni}(f_{BW})]]^2} + \frac{\sum_{i=1}^{N}[\mathrm{V}^{\mathrm{AM}}_{\mathrm{ni}}(\mathrm{f})\{\sin[\Theta_i + \Theta_{ni}(f_{BW})]\}]^2}{2BW\sum_{i=1}^{N}[V_{oi}\cos[\Theta_i + \Theta_{ni}(f_{BW})]]^2} \tag{16}$$

where $G_i(f)$ is the servo gain. Since $V^{\mathrm{PM}}_{\mathrm{refi}}(f)$ is coherent (and approximately the same) for sources i = 1 to N, this contribution will not average away when the N sources are combined in an ideal summer. Equation (16) can be simplified to the following approximate expression where we have replaced the summations by N times the average values and approximated $\cos^2[\Theta_i + \Theta_{ni}(f_{BW})]$ by 1

$$L^{Sum}_{i=1\ \mathrm{to\ N}}(f) \cong \frac{1}{2}\left[\left\langle\left|\frac{G(f)}{1+G(f)}\right|^2\right\rangle [V^{PM}_{ref}(f)]^2 + \frac{\left\langle\left|\frac{1}{1+G(f)}\right|^2\right\rangle \left\langle \mathrm{V}^{\mathrm{PM}}_{\mathrm{ni}}(\mathrm{f})\right\rangle^2}{BWN\langle V_{oi}\rangle^2} + \frac{\left\langle \mathrm{V}^{\mathrm{AM}}_{\mathrm{ni}}(\mathrm{f})\{\sin[\Theta_i + \Theta_{ni}(f_{BW})]\}\right\rangle^2}{BWN\langle V_{oi}\rangle^2}\right]. \tag{17}$$

Rewriting this in terms of single sideband PM and AM noise this becomes

$$L^{Sum}_{i=1\ \mathrm{to\ N}}(f) \cong \left\langle\left|\frac{G(f)}{1+G(f)}\right|^2\right\rangle L_{ref}(f) + \frac{1}{N}\left\langle\left|\frac{1}{1+G(f)}\right|^2\right\rangle \langle L_i(f)\rangle + \frac{1}{2N}\left\langle S_a(f)\{\sin^2[\Theta_i + \Theta_{ni}(f_{BW})]\}\right\rangle, \tag{18}$$

where $\left\langle\left|\frac{\mathrm{G(f)}}{1+\mathrm{G(f)}}\right|^2\right\rangle$ is the average PLL transfer function for imposing the PM noise of the multiplied reference on the individual sources, and $S_a(f)$ is the average PSD of AM noise for the individual sources. Equations (16-18) explicitly show the contribution of the common multiplied PLL reference source does not average with multiple sources.

For example let's consider using a second order loop with the characteristics shown in Table 1 to phase lock 100 sources operating at 10 GHz to a common reference obtained from multiplying a 10 MHz quartz oscillator to 10 GHz, using the data for possible sources given in Table 2. A block diagram is shown as Case 1 in Fig. 2. Using equation (6) we can determine the lowest offset frequency where the RMS phase deviation is less than 0.2 rad. The columns labeled Sapphire, YIG, and 10 MHz quartz show generic numbers for these types of devices.

The columns labeled YIG + 10 MHz quartz show the approximate values for locking each of 100 YIG oscillators to a common 10 MHz quartz with a BW of 300 Hz, the lowest bandwidth for the PLL and still maintain an average phase deviation due to the YIG noise of less than 0.2 radians. Table 2 also shows the results for L(f) Ave with BW = 20 kHz. The data of Table 2 shows that using a BW of approximately 300 Hz is sufficient to maintain carrier coherency between the 100 sources and therefore their PM noise should average as 1/N for f larger than roughly 100 Hz. The resulting average PM noise is however, much larger than what could be achieved using a BW = 20 kHz. With BW = 20 kHz, the averaged PM noise follows that of the single 10 MHz oscillator put to roughly 20 kHz and then gradually improves by 20 dB for f of roughly 1 MHz. These results along with the open loop noise of a generic 10 GHz YIG are shown in Fig. 3. The reason it takes such large offset frequencies to see the improvement is that the reduction of coherent contribution from the 10 MHz falls only slightly faster than the inherent YIG PM noise, using the assumed second order PLL.

Another approach would be to have each YIG locked to its own independent 10 MHz reference with a BWµw of 20 kHz and that the multiple 10 MHz sources phase locked to a common 10 MHz reference with a BW_{LF} of 1 Hz. A block diagram is shown as Case 2 in Fig. 2. The approximate results for this approach are shown in the columns labeled YIG + 100 10 MHz quartz and shown in Fig. 3. We note that the PM noise of L(f) Ave is substantially better than that of the previous configurations for f between 3 Hz and 100 kHz. This is because we have reduced the PM noise at small f by using separate 10 MHz references for each YIG while still maintaining the independent nature of the PM noise for f larger than 1 Hz.

VI. SIMPLE MODEL FOR AM NOISE FROM COMBINED SOURCES

Following the development of the PM noise of the combined sources, the AM noise of the combined sources is given by equation (19) in the limit that the phase deviation between the individual sources and the multiplied reference is small. Note that there is no term originating from the multiplied source.

$$\frac{1}{2}S^{Sum}_{ai=1\text{ to N}}(f) = +\frac{1}{2}\sum_{i=1}^{N}\left[\frac{V_{oi}}{V_{Sum}}\right]^2\left\langle S_{ai}(f)\right\rangle + \sum_{i=1}^{N}\left[\frac{V_{oi}}{V_{Sum}}\right]^2\left\langle L_i(f)\{\sin^2[\Theta_i+\Theta_{ni}(f_{BW})]\}\right\rangle . \quad (19)$$

Figure 4 shows an experimental verification of this result for 0 phase difference. Figure 5 shows the phase dependence of Eq. (15), which is similar to Eq. (19), as a function of the phase difference between Source #1 and the reference. For the special case with equal amplitudes, $V_{Sum} = N\,V_{oi}$, this becomes

$$\frac{1}{2}S^{Sum}_{ai=1\text{ to N}}(f) = +\frac{1}{2N^2}\sum_{i=1}^{N}\left\langle S_{ai}(f)\right\rangle + \frac{1}{N^2}\left\langle L_i(f)\{\sin^2[\Theta_i+\Theta_{ni}(f_{BW})]\}\right\rangle , \quad (20)$$

VII. CONCLUSIONS

A simple analytical model has been used to compute the PM and AM noise of a signal obtained by combining the output of N, nominally independent. This model takes into account different levels of noise and signal amplitude. When the phase difference between the various sources is smaller than 0.1 rad and the amplitudes of the sources are within roughly 3 dB in power, the PM and AM noise of the combined signal is 1/N of the random noise of the average source. This model shows that phase deviations between the combined signal and the individual sources leads to cross coupling between PM and AM noise. These phase deviations can originate from both static errors and noise in the sources and the PLL and reference. PM noise in the reference is impressed on the individual sources through the PLL circuits. This noise component is coherent across the sources and does not average away as 1/N. Thus the close to carrier PM noise is dominated by reference noise and PLL effects.

We find no plausible evidence for an intrinsic dependence of PM or AM noise reduction on spectral noise type in linear systems. This analysis assumes that environmental induced AM and PM noise are less than 1/3N below the noise in the individual sources and hence can be ignored.

VIII. ACKNOWLEDGMENTS

I thank Salvador Talisa, Amitave SenGupta, and Dave Howe for helpful discussions.

IX. REFERENCES

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Table 1a Idealized affect of second order PLL with a BW of 300 Hz on coherent part of 10 MHz noise in the PM noise of the individual YIGs and the reduction of free running YIG PM noise

Offset Frequency	Reduction of 10 MHz PM noise in YIG PM output $\left\langle \left\| \frac{G(f)}{1+G(f)} \right\|^2 \right\rangle$	Reduction of free running YIG PM noise $\left\langle \left\| \frac{1}{1+G(f)} \right\|^2 \right\rangle$
Hz	dB	dB
1	0	-90
3	0	-70
10	0	-50
30	0	-30
100	0	-10
300	0	0
1 k	-10	0
3 k	-30	0
10 k	-50	0
30 k	-70	0

Table 1b Idealized affect of second order PLL with a BW of 20 kHz on coherent part of 10 MHz noise in the PM noise of the individual YIGs and the reduction of free running YIG PM noise

Offset Frequency	Reduction of 10 MHz PM noise in YIG PM output $\left\langle \left\| \frac{G(f)}{1+G(f)} \right\|^2 \right\rangle$	Reduction of free running YIG PM noise $\left\langle \left\| \frac{1}{1+G(f)} \right\|^2 \right\rangle$
Hz	dB	dB
100	0	-83
300	0	-63
1 k	0	-43
3 k	0	-23
10 k	0	-6
20 k	0	0
30 k	-4	0
100 k	-18	0
300 k	-38	0
1M	-58	0

Table 2. PM noise and phase deviation of sources as a function of PLL Bandwidth.

	YIG		10 MHz Quartz		100 YIG + 10 MHz Ref			100 (YIG+10 MHz) + 10 MHz Ref	
Offset Freq.	L(f) open loop	$\Theta_{ni}(f_{BW})$ @ BW=f	L(f) open loop	$\Theta_{ni}(f_{BW})$ @ BW=f	~L(f) BWμw =300 Hz	~L(f)Ave of 100 YIGs	~L(f)Ave of 100 YIGs BW 20 kHz	~L(f) BWμw=20 kHz BW_{LF}=1 Hz	~L(f)Ave of 100 YIGs/10 MHz
Hz	dBc/Hz	rad	dBc/Hz	Rad	dBc/Hz	dBc/Hz		dBc/Hz	dBc/Hz
1	+30		-50	0.011	-50	-50	-50	-50	-50
3	+15		-65	0.01	-55	-65	-65	-65	-85
10	0	3	-80	0.01	-50	-70	-80	-80	-100
30	-15	0.9	-95	0.01	-45	-65	-95	-95	-115
100	-30	0.3	-100	0.01	-40	-60	-100	-100	-120
300	-45	0.09	-103	0.01	-45	-65	-103	-103	-123
1000	-60	0.03	-105	0.01	-60	-80	-105	-105	-125
3000	-75		-110	0.01	-75	-95	-110	-110	-130
10 k	-90		-110	0.01	-90	-100	-110	-110	-130
100 k	-120		-110	0.01	-120	-140	-128	-120	-150
1 M	-140		-110		-140	-160	-160	-140	-160
10 M	-160		-110		-160	-180	-180	-160	-180
100 M	-165		-110		-165	-185	-185	-165	-185
1 G	-185		-110		-185	-205	-205	-185	-205

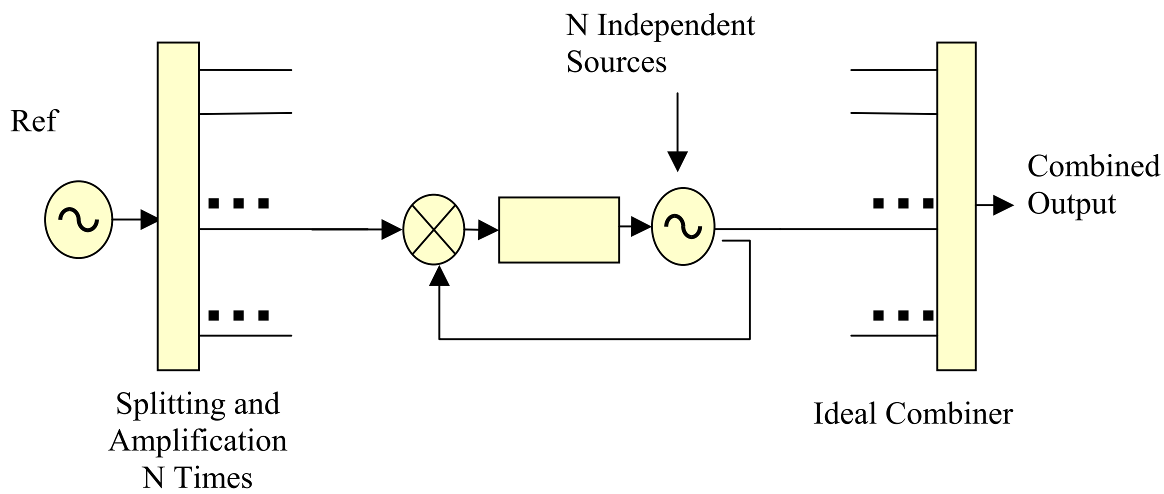

Figure 1. General Block diagram for combining multiple sources. The outputs phase of each source must be closely phase locked to the sum of the other sources.

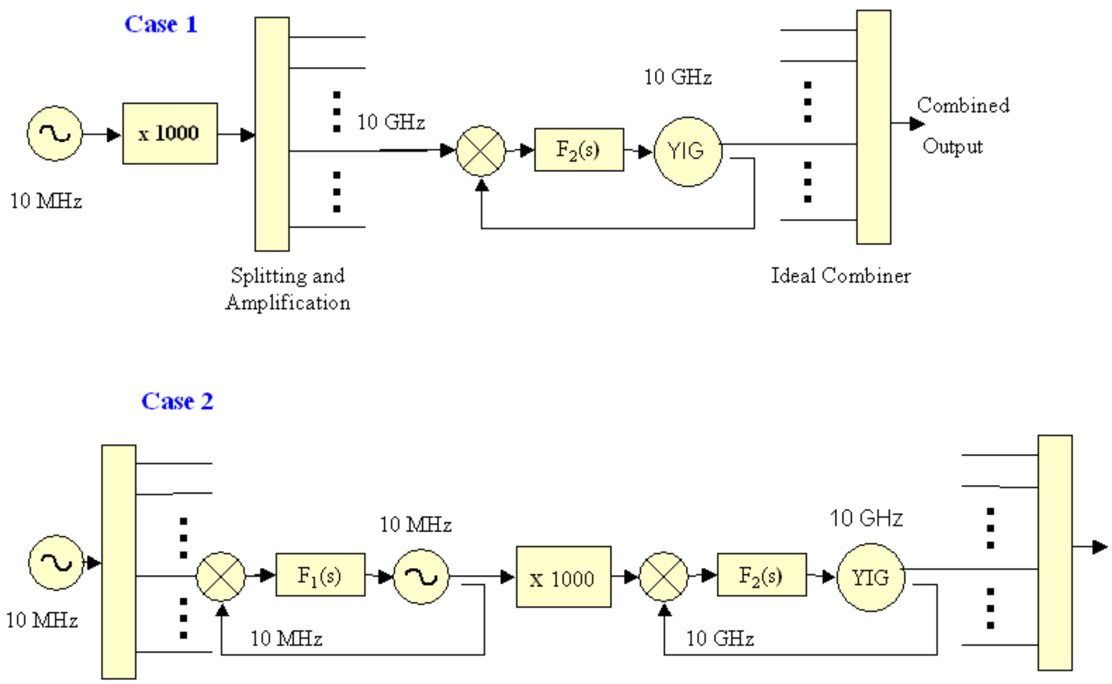

Figure 2. Block diagrams for two approaches to combine multiple 10 GHz YIG oscillators.

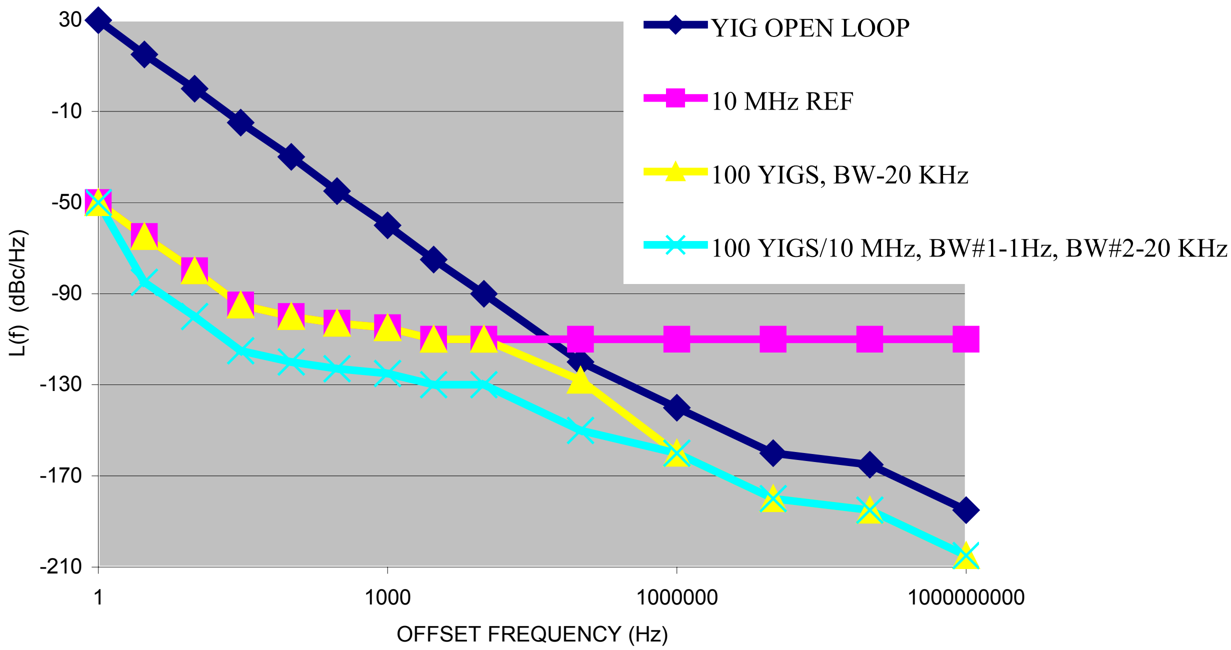

Figure 3. Effect of combing multiples sources on PM noise of output using the block diagrams shown in Fig. 2 and the PM noise of Table 2.

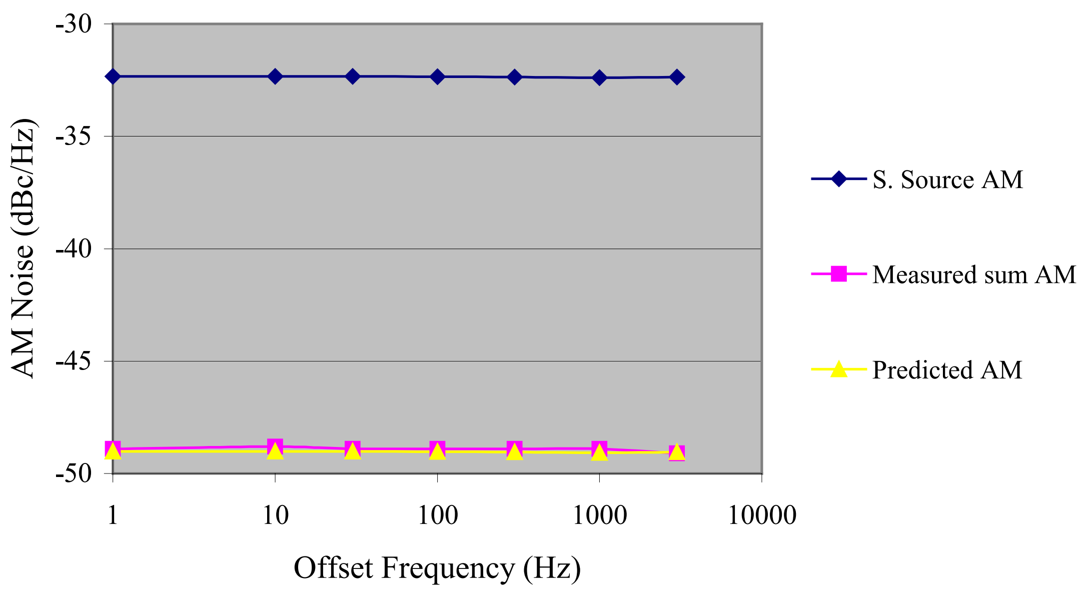

Figure 4. Measured AM of combined output due to 10% AM on source #1 when $V_{Sum} = 6.8\ V_1$ and the AM noise of the other sources is negligible. The predicted curve comes from Eq. (19). Note that there is no dependence of offset frequency f.

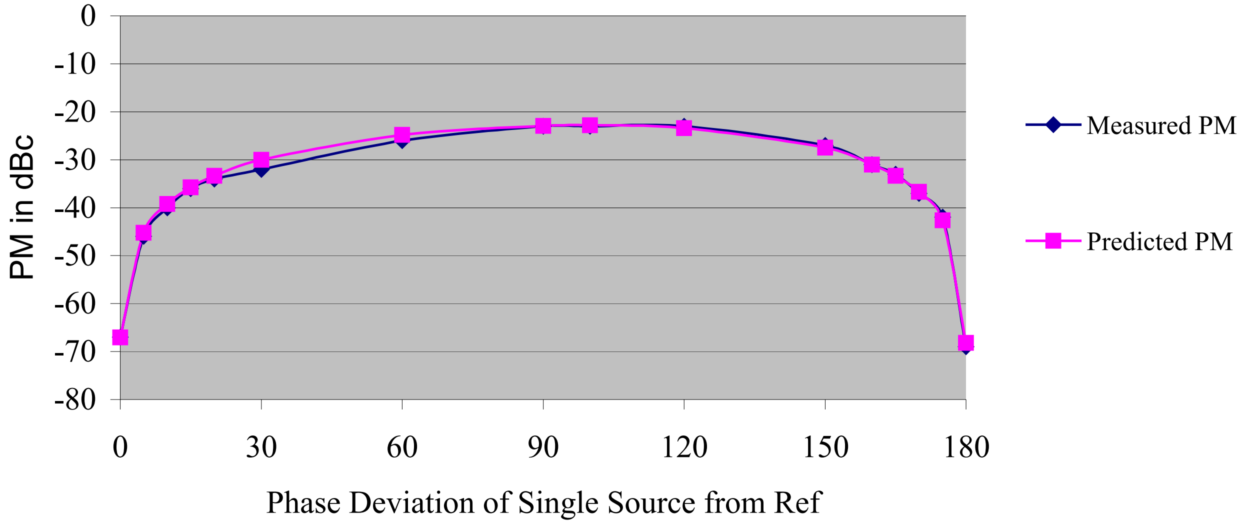

Figure 5. Measured PM noise of combined output due to AM on Source #1 when $V_{Sum} = 6.8\ V_1$ and the AM noise of the other sources is negligible. The predicted curve comes from Eq. (15).